1.

range space: span of the column vector

null space all x that satisfies Ax = 0

prove the following properties:

1. contains 0 vector
2. closed under scalar multiplication
3. closed under addition

b

i)

range(A) = {[1,0,-1,2]^T,[-1,1,0,1]^T}

null(A^T) = {[1,1,1,0],[2,3,0,-1]}

ii)

dim(range(A)) = 2 plane

dim(range(A^T)) = 2 plane

dim(null(A)) = 1 line

dim(null(A^T)) = 2 plane

iii)

range(A) orthgonal to null(A^T)

range(A^T) orthgonal to null(A)

Just show that the basis orthogonal to the basis of the corresponding space

c

i)

Just show that the equation can’t be solved.

ii)

Use least square method normal equation.

2

i)

trace(A) = 18 which is the sum of all eigenvalues

det(A) = 162 which is the product of all eigenvalues

ii)

Use part i we solve for the two eigenvalues which give us 6 and 3

iii)

We calculate (A-9I)e1 = 0 give us [2,-2,1]^T and after normalizing, give us [2,-2,1]^T / 3

Similarly for other eigenvectors, [2,1,-2]^T/3 and [1,2,2]/3

iv)

Q = [[2,-2,1],[-2,1,2],[1,-2,2]] / 3

Q inverse is the transpose of Q (since symmetric)

v)

the basis is the eigenvector calculated in iii and the diagonal matrix is diag(9,6,3)

vi)

positive definitiveness: for all non-zero vector x x^T A x > 0

x^T A x

= x^T(QQ^T) A (QQ^T) x

= (Q^T x)^T (Q^T A Q) (Q^T x)

= (Q^T x)^T B (Q^T x)

where B is the diagonal matrix.

The same can be found in Abbas’ slide.

vii)

Yes, since it’s positive definitive. And the diagonal elements should all be positive.

b

i

V = [[1,0], [0,1]]

U = [[1/sqrt(2), 0, 1/sqrt(2)], [0, 1, 0], [-1/sqrt(2), 0, 1/sqrt(2)]]

S = [[sqrt(2), 0], [0, 1] ,[0, 0]]

ii

singular values are sqrt(2) and 1. l\_2 norm is sqrt(2)

iii)

basis are [1,0] ^ T and [0,1]^T

basis are [1/sqrt(2) , 0, 1/sqrt(2)]^T [0,1,0]^T [-1/sqrt(2), 0, 1/sqrt(2)]^T

3

i)

see Abbas slides

ii)

A^-1 = [[-2,2,6],[2,3,-1],[6,-1,-3]] / 10

cond(A) = 1 \* 4 = 4

b)

d(x,x) = 0

d(x,y) > 0 if x /= y

d(x, y) = d(y,x)

d(x,y) <= d(x,z) + d(z,y)

And we can prove the above using the properties of norm.

For details see tutorial 6 questions 7

4

part a is all book work

for a iii notice that the L(-s) is the mgf of the probability function. And the result follows.

b ii)

y1(t) = ⅔ e^(-t) + ⅓ e^(2t)

y2(t) =- ⅔ e^(-t) + 1/6 e^(2t)